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Testing for Extragalactic Correlations of Gamma-Ray Bursts

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ABSTRACT

If gamma-ray bursts (GRBs) are cosmological they need not necessarily be spatially coincident with a host galaxy—in many cosmological models the progenitors are ejected to large distances from their parent galaxies. While the optical transient of GRB 970228 may be close to a faint galaxy, there are several other galaxies in the field which could be the host if the progenitors are ejected to $\sim 50 h_{70}^{-1}$ kpc. If GRBs come from such a widely distributed progenitor population then it should be possible to search for a statistical correlation between optical transients and nearby galaxies. We provide a statistical framework for quantifying a correlation between potential host galaxies and GRBs, taking into account that galaxies are clustered. From simulations, we estimate that if GRBs occur within $l \lesssim 100$ kpc of their host, then optical observations ($B \leq 24.0$) of the galaxy field near just 6 optical transient positions are required to find a correlation with potential hosts at the 95 percent confidence level. The methodology constructed herein is extendible to any cross-correlation analysis and is especially useful if either set of objects is clustered. Ultimately, if the distance scale to GRBs is established as cosmological, this analysis can be used to determine the spatial distribution of GRBs near their host galaxies.

Key words: gamma-ray bursts—faint galaxies—correlation statistics.

1 INTRODUCTION

After nearly thirty years since the discovery of gamma-ray bursts (GRBs), a fading afterglow has been detected in X-ray and optical wave-bands (Van Paradijs et al. 1997). This observational breakthrough has, for the first time, provided both a precise localisation of a GRB position and broad-band spectral information. But, while the spectral fit by Wijers, Rees & Mészáros (1997) does agree quite well with the cosmological blastwave theory (Rees & Mészáros 1992), a coherent picture of the distance scale to GRBs has yet to emerge. Preliminary results as to the nature of the observed field surrounding the optical transient position have been inconclusive: a faint extended source near the optical transient position may have faded from images taken at Keck at two different epochs (Metzger et al. 1997a; although see Fox et al. 1997). The natural assumption was that if GRBs are cosmological in origin then the optical transient would be associated with a galaxy; if the extension is indeed variable, then this is not the case. Furthermore, a spectrum of the optical transient and the extension taken with the Keck II telescope (Tonry et al. 1997) does not reveal any tell-tale features which would indicate the distance to GRB 970228. So far no extended emission has been detected from the recent burst GRB 970508 but Metzger et al. (1997b) have reported absorption lines in the spectra of the optical transient putting the transient beyond $z = 0.835$. If confirmed, this would establish GRBs as cosmological.

If GRBs are cosmological, what are the other likely observational consequences which would rule out the non-cosmological scenarios? One answer is that in most cosmological models GRBs will have a parent galaxy which we can search for. Historically this has been difficult due to the scarcity of small error boxes: the Interplanetary Network (IPN) of satellites yielded only a few error boxes smaller then about 5 arcmin^2 (Atteia et al. 1987). As the projected density of galaxies is reasonably high (eg. we expect about 30 galaxies in 5 arcmin^2 boxes at $B < 24.0$) there was no lack of potential hosts galaxies to the bursts; statistical studies of GRB positions were thereby limited to searching for correlations on scale lengths greater than several arcmins (Kolatt & Piran 1996; Hurley et al. 1997).

Kolatt & Piran 1996, for instance, do find evidence of a correlation of bursts with Abell clusters using the crude BATSE localisations (although see Hurley et al. 1997). Using the better, but fewer, localisations from the IPN, there

have also been claims of a bright galaxy excess near bright bursts (Larson, McLean, & Becklin 1996) but this analysis uses Poissonian errors to quantify over-density, neglecting the fact that galaxies are clustered and counts far galaxies outside the strict limits of the IPN error boxes. Fenimore et al. (1993), examining 8 of the smallest IPN error boxes, conclude that host galaxies of GRBs must be fainter than an absolute visual magnitude $M_v \simeq -18$. Recent *Hubble Space Telescope* (HST) observations of IPN boxes confirm this result (Schaefer et al. 1997) and further constrain the host galaxies of GRBs to be fainter than absolute $B_{mag} \gtrsim -17.4$. The conclusion that this requires the hosts of GRBs to be significantly fainter than L_* is valid only if GRBs are standard candles and if the faintest BATSE bursts have $z \lesssim 1$. Moreover, it does not allow for the possibility that a galaxy outside IPN error boxes could be the host to the GRBs.

With the detection of the optical transient to GRB 970228 we now have the beginning of a set of very high precision positions and correlation analysis should thus begin to probe smaller scales at deeper magnitudes. Deep HST imaging reveals faint extended emission adjacent to the 970228 optical transient point source. If further observations prove that this is a distant galaxy, then it must be a very good candidate for the host of the GRB. However, at a magnitude of $M_{F606W} \simeq 25.5$ (Tanvir & Johnson 1997) the spatial density of galaxies is high and the alignment could be a chance coincidence. Thus Galactic models are not ruled out and neither are cosmological models in which the progenitors of GRBs are ejected from their host or even only be loosely associated with the light density from galaxies. If GRBs arise from neutron star–neutron star (NS–NS) coalescence, for instance, then a GRB may occur tens of kpc outside of its host galaxy (Sigurdsson, private communication); at redshifts of $z \sim 0.2$, such positional displacements allow an offset of the optical transient from its host galaxy by as much as 20 arcsec. If GRBs do occur far outside their hosts then unambiguously identifying the parent galaxies will be difficult. However, to rule out the Galactic scenario, it would suffice to find a statistically significant over-density of potential host galaxies near GRB positions since bursts originating from within our Galaxy should be randomly distributed with respect to underlying galaxy field.

Clearly, to distinguish between the two distance scales, there must be a method which can both determine an over-density of potential hosts (if there is one) and accurately estimate the significance of such an over-density. Ultimately, if the distance scale to GRBs is established to be cosmological, then the same analysis will help reveal how GRBs are distributed near galaxies.

In section 2 we provide a framework for making quantifiable statistical statements about the degree of over-density of hosts for a given number of GRB positions that accounts for galaxy clustering. In section 3 we describe the simulation of a galaxy distribution which is used in section 4 to predict the number of GRB positions needed to find a significant correlation if GRBs are indeed associated with distant galaxies. In section 5 we show how the method developed herein may be extended to other types of data and hypotheses. In other words, should observations provide no evidence for a positive correlation, the test developed in section 2 can be used to accept the alternative hypothesis that GRBs *are not* correlated with potential hosts to faint limits (obviously, by rejection of the null hypothesis that there *is* a correlation under some given set of assumptions). We consider this issue in light of results from HST images of GRB 970228.

2 CONSTRUCTION OF A STATISTIC OF OVER-DENSITY

2.1 What is a potential host?

In an overwhelming number of viable cosmological models, a GRB originates from near or within a galaxy. In each model scenario, there is a three-dimensional probability, $P_3(\vec{x})$, of a GRB originating from a given galaxy at a co-moving position, $\vec{x}$, relative to its centre. In the simplest case, we take the probability of a GRB occurring to be constant within a sphere of radius l. The projected two-dimensional probability distribution on the sky, for a galaxy at redshift z is thus:

$$P_2(l, \theta, z) d\theta = \frac{3A_i(z)\sqrt{D^2 - \theta^2} d\theta}{2\pi D^3}, \qquad (1)$$

where

$$D = \frac{lH_0}{2c} \frac{(1+z)^2}{[(1+z) - (1+z)^{1/2}]} \text{ radians} \qquad (2)$$

in the standard, $q_o = 1/2$, $\Lambda = 0$, cosmology. Hereafter, the Hubble constant is taken as $H_0 = 70$ km s^{-1} Mpc^{-1} (eg. Tanvir et al. 1995). For each burst, i, there is a total probability normalisation, $A_i(z)$, for each galaxy in the field. If the results of the log N-log P studies are to be used as a guideline (as in Fenimore et al. 1993), then a bright GRB—such as the expected case for most BeppoSAX X-ray detections or small IPN error box localisations—is expected to come from a low-redshift galaxy and thus $A_i(z)$ for that particular burst will peak at a low redshift.

For each burst position i (which is observed in a given passband, and limiting magnitude) one constructs a probability host map, PHM$_i(r)$, by adding the contribution of $P_2(r, \theta, z)d\theta$ from each galaxy where r is the maximum offset scale of the map. A PHM gives the relative probability of a GRB occurring at any position on the sky assuming a particular functional form of the offset probability, $P_2(r, \theta, z)$. If GRBs are not correlated with galaxies then their positions should occur randomly on the PHM; conversely, if GRBs are correlated with galaxies then their positions will arise in regions of larger probability density on the PHM.

Since we do not know *a priori* both the intrinsic offset scale l and the functional form of the offset probability distribution, we must assume some map scale r and a form of the offset probability distribution [$P_2(r, \theta, z)$]. In addition, construction of the PHM requires knowledge of the redshift of each galaxy; redshifts may be determined from photometric colours, via their observed spectra, or simply estimated from their brightness using previous flux/redshift studies. Alternatively, a median redshift of galaxies in the field may be assigned to each galaxy. In the simulations in section 3, we show that, not surprisingly, a PHM is most sensitive to detecting correlations if $r \simeq l$. We also find that assigning the median redshift to each galaxy in the map suffices in finding correlations (see fig. [3]) at least for small intrinsic offset scales. Note that if the map scale r is sufficiently small (i.e. if GRBs are retained within a few kpc of a host galaxy),

the PHM reduces to a series of δ-functions. In section 6 we discuss what a reasonable value for l might be for different models.

In what follows, let $\rho_{i,j}$ be the value of $\mathrm{PHM}_i(r)$ for the jth randomly selected location on map i; by counting the frequency of $\rho_{i,j}$ for a large number of randomly selected points we construct the distribution $f_i(\rho)$, which is effectively the probability of finding ρ of a potential host on the sky at any point on the map.

2.2 The Test for Over-densities of the True Sample

For a given passband and limiting magnitude of an observation of each position, there will be a corresponding number density, $\bar{N}_i$, and two-point correlation function, $\omega_i(\theta)$, that characterises the underlying distribution of galaxies from which $f_i(\rho)$ is found. In section 3 we simulate an underlying galaxy distribution that matches the observed properties. In practice, it may be better to estimate $f_i(\rho)$ directly from a number of control fields taken at similar airmass, galactic latitude, limiting magnitude, etc.

The normalised cumulative probability function is given as:

$$F_i(\rho) = \frac{\int_{\rho_{\min,i}}^{\rho} f_i(\psi)d\psi}{\int_{\rho_{\min,i}}^{\rho_{\max,i}} f_i(\psi)d\psi}, \tag{3}$$

where $\rho_{\min,i}$ and $\rho_{\max,i}$ are the extreme values of ρ on a given PHM_i. If $\rho_{i,\mathrm{obs}}$ is the observed value of ρ on the PHM of burst i then the set of numbers $S = \{F_i(\rho_{i,\mathrm{obs}})\}$ will be evenly distributed in the interval [0,1] if the positions of the GRB ensemble are uncorrelated with that of the potential host galaxy distribution.

The degree of deviation of S from an evenly distributed, cumulative set can be used to estimate the strength of the correlation between GRBs and observed potential host galaxies. Thus, this method allows one to combine an inhomogeneous set of observations to test the hypothesis that GRBs are correlated with host galaxies.

The significance of the discrepancy between the observed set S and the expected set is evaluated using the Wilcoxon statistic (see Noether 1967 and Efron & Petrosian 1996). It is defined as:

$$W = \sqrt{12 N_{\mathrm{burst}}} \left[0.5 - \frac{1}{N_{\mathrm{burst}}} \sum_{i=1}^{N_{\mathrm{burst}}} F_i(\rho_{i,\mathrm{obs}}) \right], \tag{4}$$

and compares the average value of S and the expected value of the cumulative probability distribution ($\equiv 0.5$). *The distribution of W values, $f(W)$, for an uncorrelated set of GRB positions and potential hosts should be normally distributed with variance unity centred about zero.* In section 3 we construct $f(W)$ for a given limiting magnitude and number of optical transient source positions by a Monte Carlo simulation of many expected (uncorrelated) sets (see fig. [2]). We test the null hypothesis that GRB optical transient source positions are uncorrelated with host galaxies by comparing the stochastic realization of W_{obs} with the expected distribution of $f(W)$. Clearly if there is a positive correlation, then the W-statistic will be negative (and the opposite is true for a anti-correlation). In this formalism, the null hypothesis can be rejected with a quantifiable confidence level by comparison of W_{obs} with $f(W)$.

3 PREDICTIONS

In this section we construct simulations which will be used to assess the power of the above analysis under different assumptions about the true GRB distribution. Specifically we will use these simulations to ask how many observations would be required to show some correlation of GRBs with galaxies in various scenarios.

3.1 Constructing a Probability Host Map (PHM)

To estimate the expected significances, we construct a galaxy distribution which artificially replicates the observed galaxy number density, two-point correlation function, and redshift distribution for a given limiting magnitude. Several galaxy distribution studies have provided the necessary data to construct such a distribution but here we limit ourselves to the correlation/redshift study of Roche et al. (1996). For a given limiting magnitude m in the R- and B-band, they provide a redshift distribution of galaxies ($f_m(z)$), number density ($\bar{N}_m$), and fit to a two-point correlation function ($\omega_m(\theta)$). The observations were made at high galactic latitude ($b \simeq 85^\circ$) and with little extinction ($E_B \simeq 0.15$ and $E_R \simeq 0.08$). This simulation is obviously extendible using other galaxy distribution studies which provide similar data in other passbands and/or limiting magnitudes.

3.2 Simulating a PHM from galaxy spatial and redshift distribution

As mentioned earlier, it may be possible to construct a PHM directly from control fields taken under similar conditions as the optical transient field. Here, however, we generate a PHM artificially. To generate an artificial galaxy distribution, we employ the truncated hierarchy method (Soneria & Peebles 1978) which has been adapted to 2-D projections (Infante 1994). For a given burst position i we vary the intrinsic scaling θ_0 of the algorithm (see Infante 1994) to fit the expected spatial distribution of underlying galaxies (quantified by the spatial density distribution, $\bar{N}_i$ and the amplitude and slope, α, of the two-point correlation function). For each galaxy in the distribution, we construct a probability profile from eq. [1] replacing l with r and thereby construct a $\mathrm{PHM}(r)$.

The construction of the galaxy distribution and PHM requires a few assumptions.

(i) We take the slope of the two-point correlation function to be $\alpha = -0.8$ which, based on numerous galaxy distribution studies (eg. Roche et al. 1996), seems reasonable. In our simulation using the truncated hierarchy method, for $B \leq 24.0$, we are able to reproduce the two-point correlation function and number density ($\bar{N} = 20730$ gals/deg^2) that match that found in Roche et al. 1996. Bright GRBs are expected to come from moderate redshifts. We have therefore chosen a limiting magnitude of $B \leq 24.0$ which corresponds to a median redshift of $z \sim 0.6$. Clearly the analysis may be extended to fainter magnitudes in which case the expected

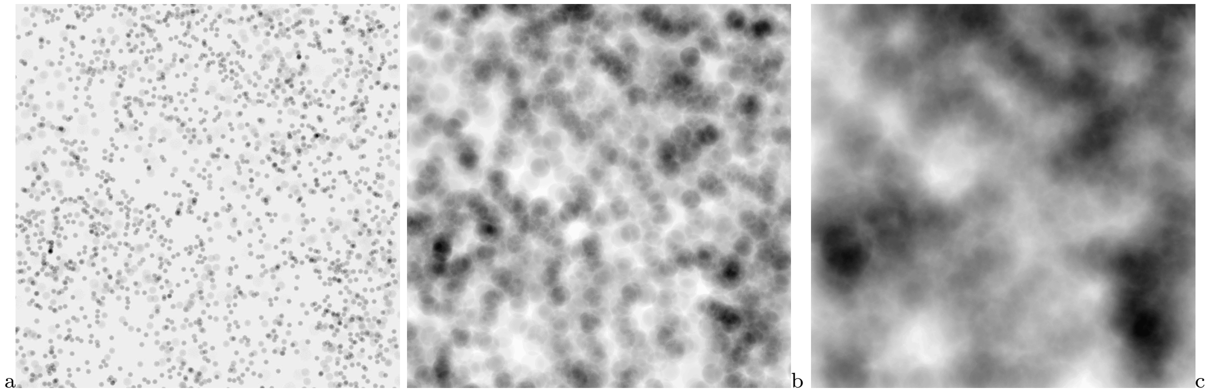


Figure 1. Simulated probability host map (PHM) from galaxy fields ($B \leq 24.0$) for a) $r = 50$ kpc, $r = 200$ kpc, and, c) $r = 500$ kpc. Regions of higher potential host density are darker and the size of the circular region around each galaxy is related to its redshift by eq. [1]. Each field, comprising about 2300 galaxies, is a 20 arcmin × 20 arcmin portion of a large PHM. The simulated clustering of galaxies evident gives a larger spread in $f(\rho)$ than that derived from a randomly distributed set of galaxy positions.

number of burst positions required to find a correlation will increase.

(ii) We assign a redshift to each galaxy in the generated distribution. Roche et al. (1996) provide an estimated galaxy redshift distribution as a function of limiting magnitude. For each cluster in an artificially generated spatial distribution, we then assign a similar redshift (±5 percent) to its constituent galaxies based on a probability distribution constructed from the Roche et al. (1996) data. This accounts for both for the observed distribution of redshifts and the fact that galaxies within the same cluster have similar redshifts.

(iii) We take $A_i(z)$ equal to unity. Note that *any* non-uniform dependency of $A_i(z)$ upon redshift will reduce the spatial density of potential hosts and thus increase the detectability of a correlation. Although the distances to GRB hosts should be in accord with distance implied by the log N-log P, it is not clear that the log N-log P gives a meaningful constraint on the functional form of $A_i(z)$ since a standard luminosity of GRBs appears to be unconstrained (Loredo & Wasserman 1996; Horvath, Mészáros, & Mészáros 1996).

A 20' × 20' portion of three PHMs is depicted in figure (1) for various map scale lengths, r. The clustering of galaxies is evident.

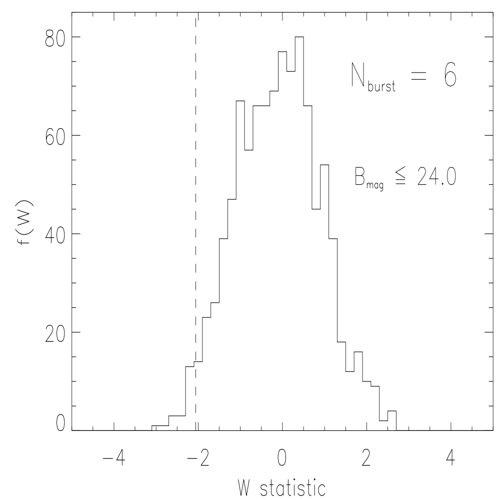


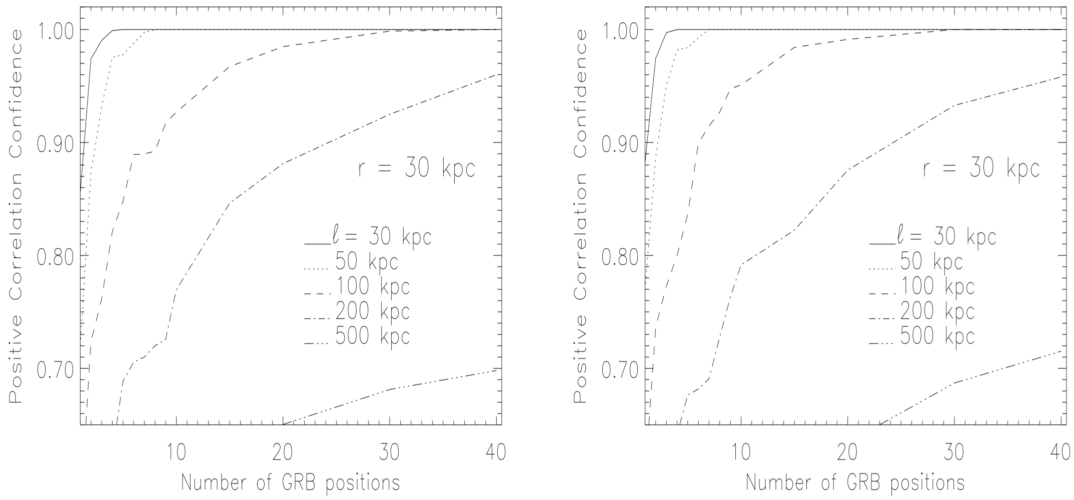
Figure 2. The expected distribution of $f(W)$ for an uncorrelated set of 6 GRB positions for $l = r = 100$ kpc. Also, shown with a vertical dashed line, is the predicted W determined from a correlated set of 6 GRB positions. From the distribution, we compute that such a value ($W \simeq -2$) or smaller could occur by random chance in only 3.1 percent of cases if the GRB positions are not correlated with potential hosts. Thus, the null hypothesis may be rejected at greater than 95 percent confidence.

4 SIMULATION RESULTS

At a given limiting magnitude, bandpass, and number of optical transient positions, $N_{\rm burst}$, we find the Wilcoxon distribution $f(W)$ as prescribed in section II. We then simulate an over-density of host galaxies near GRBs by choosing a random galaxy on the sky and placing the optical transient position at an offset prescribed by eq. [1]. The stochastic realization of $W_{\rm obs}$ vs. $f(W)$ is used to determine the significance of a correlation. Figure (2) shows an example comparison of $W_{\rm obs}$ (dashed vertical line) vs. $f(W)$. For a given limiting magnitude, map scale r, and intrinsic scale l, we predict the probability that the stochastic realization of W is consistent with the null hypothesis,

H_0: GRB positions are uncorrelated with potential hosts.

For a magnitude $B \leq 24.0$ and $r = 30$ kpc, figure (3) shows the confidence in rejecting H_0 as a function of the number of GRB positions and l. Notice that if l is less than ~ 50 kpc, then only a few GRB positions are required to rule out H_0 and conclude that GRBs are correlated with host galaxies. This is not a surprising result given that the typical projected distance at $z \sim 0.3$ for $l = 30$ kpc is only a few arcsec: the expected value of PHM($r = 30$) is close to zero and thus any over-density is rare. The correlation analysis is sensitive to the chosen map scale r and whether knowledge of galaxy redshifts are included. Figure (4) shows the expected number of positions needed to find a 95 percent confident over-density (if there is one) as a function of l and r where the PHMs were constructed using only galaxy positions and the same (median) redshift ($z \simeq 0.61$) for each



a b

Figure 3. Null hypothesis rejection confidence as a function of the number of GRB positions. The null hypothesis tested is that GRBs are uncorrelated with the potential host distribution. The PHM is made with $r = 30$ kpc and taking either a) every galaxy is located at the median redshift ($z \simeq 0.61$) of $B \leq 24.0$ galaxies (Roche et al. 1996) or b) the simulated redshift for each galaxy. The predicted curves for $l = 30$ kpc (solid), $l = 50$ kpc (dot), $l = 100$ kpc (dash), $l = 200$ kpc (dot-dash), and $l = 500$ kpc (dots-dash) are depicted; the small deviations from a smooth curve are the result of a finite number of trials in the Monte Carlo simulations. For $l \lesssim 50$ kpc, only a few burst positions are required to find a 99 percent confident correlation of GRBs with potential hosts. Note that with increasing values of l it becomes more difficult to detect over-densities. However, other chosen PHM scales r are more sensitive to correlations at larger l (see fig. [4]). The predictions do not greatly differ for each type of map. Thus, precise knowledge of the redshifts of galaxies will not be necessary to construct a PHM.

galaxy. Small values of r produce PHMs that are most sensitive to a correlation if l is also small; likewise, correlations are most readily found if l is large by a choice of a large value of r in making the map. As evidenced by the small number of bursts required to make a significant correlation, the redshift of each galaxy, inclusion of which strengthens the test for correlations, need not be precisely known in making the PHM. The simulation predictions presented in fig. [4] are made from PHMs where the same single redshift is assigned to all galaxies. Figure [3] illustrates the minor strengthening of the correlation test where knowledge of galaxy redshifts is included in construction of the PHM.

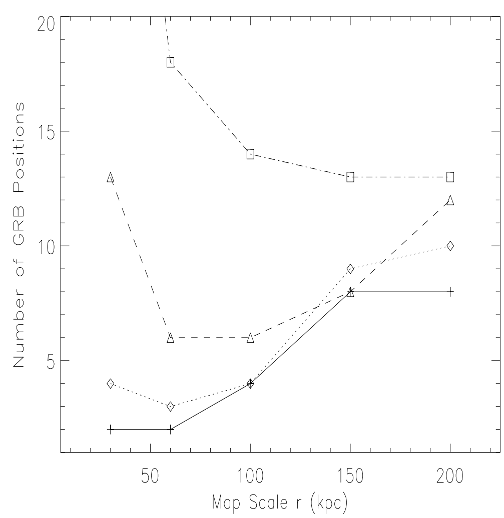


Figure 4. Number of GRB positions needed for 95 percent confident correlation detection as a function of map scale r. The intrinsic scale of GRBs in the simulation are $l = 30$ kpc (solid line), $l = 50$ kpc (dotted), $l = 100$ kpc (dashed), and $l = 200$ kpc (dot–dashed). Notice that a correlation is most readily found with $r \simeq l$.

5 EXTENSION TO OTHER DATA SETS AND HYPOTHESES

5.1 Using IPN localisations

A significant correlation of GRBs with observed galaxies may be found without further precise locations that optical transients would provide. Instead, the method described above may be used on observed fields near existing IPN localisations. In such a case, $f_i(\rho)$ is estimated by randomly selecting positions on a PHM and taking ρ to be the average density within an error box of identical shape and size as IPN localisation i. The observed value of ρ_i is then compared against $f_i(\rho)$ as above.

5.2 Are GRBs Uncorrelated with Observable Galaxies?

If the extension near the position of GRB 970228 is not a galaxy there are no hosts at the faint HST limit ($M_{F606W} \lesssim$

Table 1. Gamma-ray burst 970228 constraints on the minimum offset of GRBs from their host galaxy. The data found is from cleaned HST PC1 images taken on March 26 and April 7 (Tanvir & Johnson 1997). Instrumental magnitudes are derived in two passbands and errors are found from systematic ($\simeq 0.1$) and photometric uncertainties. Zero-point calibration is taken from Holtzman et al. (1995). Redshifts are estimated from an F606W magnitude comparison with galaxies in the Hubble deep field (Cowie 1997). The redshift at which 90 percent of the Cowie galaxies of similar magnitudes have larger photometric redshifts is denoted by z_{90} (similarly, z_{50} is the median redshift of galaxies with similar magnitudes). This corresponds to a constraint on the minimum offset l_{90} (assuming $q_\circ = 0.5$) of GRBs from their host galaxies.

Galaxy Number	Position (J2000) RA (hms)	DEC (deg)	D (arsec)	Magnitude F606W	F606W − F814W	Redshift z_{50} (z_{90})	l_{50} (l_{90}) (h_{70}^{-1} kpc)	Galaxy Property
1	5:01:46.50	11:46:50.2	4.1	26.3 ± 0.4	0.9	1.34 (0.25)	25 (14)	
2	5:01:46.20	11:46:45.4	10.8	21.8 ± 0.1	1.1	0.31 (0.09)	43 (17)	spiral, star-forming?
3	5:01:46.56	11:46:46.1	7.7	23.7 ± 0.1	0.8	0.70 (0.20)	44 (23)	
4	5:01:47.17	11:46:49.3	8.8	25.5 ± 0.21	0.8	0.88 (0.62)	53 (48)	
5	5:01:47.15	11:46:54.4	5.2	25.2 ± 0.23	1.0	0.87 (0.25)	31 (18)	emission-line galaxy
						$z = 0.638^a$	28.9	
6	5:01:46.94	11:47:04.1	11.4	25.4 ± 0.30	0.5	0.84 (0.61)	67 (62)	

[a] Spectroscopic redshift from Tonry et al. 1997.

26.5) for a GRB/host-galaxy offset of $l \lesssim 14 h_{70}^{-1}$ kpc (see table 1). If we take the less faint limits of $B \leq 24.0$ commensurate with simulations, then the somewhat stronger constraint of $l_{\min} \gtrsim 17 h_{70}^{-1}$ kpc comes from galaxy 2 in table 1. As the redshifts of the galaxies in the HST fields are yet to be determined spectroscopically, these offset lower limits should be considered approximate.

While this one optical transient position does not provide a strong constraint, it does seem that either it has a surprisingly faint host or GRBs occur at large offsets from their host galaxy. If the extension is indeed a galaxy, then it is so faint that there is a non-negligible chance (few percent) that the optical transient position is coincident with it by random chance.

The absence of a detectable galaxy down to faint magnitudes ($M_{F606W} \lesssim 25$) near the optical transient position of GRB 970228 gives sufficient impetus to the formulation of the alternative null-hypothesis that GRB positions are correlated with potential hosts. A significant rejection of this hypothesis will place important constraints on most cosmological models. Within such a hypothesis, $f(\rho)$ is constructed by choosing positions near galaxies (rather than at random) from a PHM. By examination of the HST fields taken on 26 March and 7 April, already one can rule out H_0 for $l \lesssim 17 h_{70}^{-1}$ kpc for $B \lesssim 24$ (galaxy 2; table 1). Obviously, GRBs could originate from low-surface brightness galaxies or at large redshifts, in which case the detectability of a correlation would be difficult.

6 DISCUSSION

Regardless of the interpretation of the emission near the optical transient position, it is clear that GRB 970228 did not originate from a bright, nearby galaxy (see table 1) confirming the systematic findings of Schaefer et al. (1997) and Fenimore et al. (1993). It should be emphasised that the "no-host" conclusion of Schaefer et al. (1997) and Fenimore et al. (1993) is based strongly on the assumption that the GRB population follows a non-evolving, standard candle luminosity in as much as it is argued that bright bursts should come from nearby galaxies. But from an observational standpoint, Loredo & Wasserman (1996) have concluded that the width of the luminosity function of GRBs is poorly constrained and Horvath, Mészáros, & Mészáros (1996) find that a large range of luminosities acceptably fit the log N-log P distribution. Moreover, theories of cosmological shocks and blastwaves predict a GRB luminosity which can vary greatly from burst to burst (eg. Fenimore, Madras, & Nayakskin 1996); instead total energy is more likely to be standard (see Bloom, Fenimore, & in 't Zand 1997). Thus, robust conclusions about (non-)correlations of hosts with GRBs should not rely on the above assumptions.

The specific correlation hypothesis that is tested is wholly determined by the functional form and dependences in eq. [1]. The basic hypothesis we have tested in these simulations is that "the progenitors of GRBs originate (and are possibly ejected) from observable galaxies." If mass in the Universe scales with light, this can also be seen as a test that the rate density of GRBs is highest at the peaks of the matter distribution. Of course while this hypothesis is somewhat model dependent, it effectively introduces only one free parameter, namely l (and to a lesser extent P_2).

We conservatively estimate the maximum intrinsic offset scale to be of order $l = 400$ kpc. This corresponds to a constant progenitor outward velocity $v_{\rm pro} = 400$ km/s and a collision time scale of $\tau = 10^9$ years. If all GRBs occur at such large distances from their host galaxy, it would be difficult to find a significant correlation with a small collection of GRB positions. However, simulations have shown for NS-NS models that l may be an order of magnitude smaller and $P_2(l)$ is likely to be very peaked near the centre for most galaxies, so that the effective l might be significantly lower than 400 kpc (Lipunov et al. 1995). Therefore, whatever the true l is, we believe that the assumed $P_2(r = l)$, is quite conservative by over-predicting the true extension of GRBs from their hosts.

Since l is not known it is impossible to know *a priori* which map scale r will optimise the detection of a correlation. The predictions of a 95 percent confident correlation after about six optical transient detections is based on a fixed $r = 60$ kpc and assuming $l \leq 100$ kpc (see fig. [4]). Although

a detected correlation thusly will be adequate for resolving the distance scale to GRBs debate, ideally one would like to determine l in addition. As a correlation search over a range of map scales r is likely to increase the random chance of finding a correlation, it will be necessary to determine l *a posteriori* via Bayesian analysis.

One limitation of the statistical method presented is that the angular size of the PHM must be significantly larger than the projected angular offset D of GRBs from their host. As most cosmological models would have GRBs originate at distances significantly less than 1 Mpc from their host galaxy, this corresponds to a minimum PHM size of about 30 arcmin $\times$ 30 arcmin. The correlation analysis hinges on an accurate modelling of the expected probability density distribution ($f(\rho)$) and thus control fields must be selected quite carefully in order not to bias the PHM. We will examine the salient issues of estimating $f(\rho)$ in a future paper, which presents the results of a deep optical IPN error box correlation analysis.

7 CONCLUSIONS

We have presented a formalism which may be used to quantify the degree of over-density of potential host galaxies near GRB positions; if GRBs are found to be cosmological in origin this formalism may also be used to determine the intrinsic offset of GRBs from their hosts. The broad hypothesis we have chosen to address is that GRBs originate within l kpc of their host galaxy. In our simulations, we find that for $l \lesssim 100$ kpc, encompassing most predictions of current cosmological models, a 95 percent significant correlation may be found with about six positions. This prediction corresponds to simulations of galaxy fields of $B \leq 24.0$ where the median redshift of galaxies is $z \sim 0.6$. If GRBs are correlated with galaxies at comparable redshifts, such correlation may found within a year if BeppoSAX detections lead us to a handful of optical transient localisations.

However, the extension near the optical transient of GRB 970228 is very faint ($M_{F606W} \simeq 25.5$ implying a redshift $z \sim 1$), which suggests that GRBs may only correlate with very distant galaxies. In this case, correlations will be detectable only if l is sufficiently small. If the fading extension near the optical transient of GRB 970228 is not a galaxy, we find that the progenitor system must have travelled a minimum distance $l_{\min} \gtrsim 14h_{70}^{-1}$ kpc from its host galaxy before producing a GRB. Assuming that galaxy detection in the HST exposures is complete to distances of the bright burst, this effectively requires GRBs to originate well outside the nucleus of their host galaxies. In any event, the absence of a bright host galaxy indicates that the non-evolving standard luminosity derived from the log N-log P distribution inadequately describes the true distribution of cosmological GRBs.

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